Determination of Orthometric Heights by GNSS Observations through an Accurate Geoid Undulation Model for Sri Lanka

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Abstract

This study presents the development of a Geoidal Undulation Model for Sri Lanka, aiming to enhance the accuracy of orthometric height calculations derived from GNSS observations. Orthometric height, which reflects true elevation above the geoid and accounts for variations in the Earth's gravitational field, is crucial for applications in construction, surveying, and infrastructure development. Traditional methods, such as differential leveling, are time-consuming and less efficient compared to GNSS surveying. This research addresses the need for an effective way to convert ellipsoidal heights obtained through GNSS into practical orthometric heights. To achieve this, geoidal undulation data from nearly 750 strategically distributed control points were analyzed using a polynomial surface fitting method, complemented by the least-squares adjustment for model parameter determination. The model was checked for the same data points in the model and for a independent data set which was consisted 72 data points. The findings of geoid model was revealed that 64.1% of the modeled dataset and 61.3% of independent dataset achieved an accuracy of ± 0.30m when compared to orthometric height determined through leveling techniques. The model was further developed by integrating gravity measurements from EGM2008 to develop a hybrid geoid model, which enhanced the accuracy and reliability of the geoidal undulation model for Sri Lanka. The result of the hybrid model was verified that 96.1% of the modeled dataset and 98.6% of independent dataset achieved an accuracy of ± 0.30m compared to orthometric height determined through leveling techniques. These results were underscored the model's potential to streamline orthometric height calculations, making GNSS observations more applicable for real-world applications such as topographical mapping, engineering surveying etc. In future, the accuracy and the reliability of the hybrid model will be increased by incorporating terrestrial gravity data.

Keywords: Geoid undulation, Hybrid Geoid model, Least-square adjustment, Orthometric height, Polynomial Fitting

Introduction

a. Background

The Earth's shape and gravitational field are more complex than the simple models of spheres or ellipsoids might suggest. This complexity is captured by the concept of the geoid, an equipotential surface representing the mean sea level in the absence of external forces like winds and tides.

Geoid

As it is described previous sub section, the geoid is a model of the Earth's mean sea level, which represents the shape that the surface of the oceans would take under the influence of Earth's gravity and rotation, ignoring other influences like winds and tides. The geoid is an equipotential surface of the Earth's gravity field, meaning that at every point on the geoid, the gravitational potential is the same. This characteristic allows the geoid to be a reference surface for measuring gravity-related phenomena. Unlike the smooth, idealized shape of an ellipsoid, the geoid is irregular. This irregularity arises due to variations in the Earth's gravitational field caused by factors such as the distribution of mass within the Earth, the density of rocks, and the topography of the sea floor. These variations cause the geoid to rise and fall relative to a reference ellipsoid.

Global Ellipsoid

A global ellipsoid, also known as a reference ellipsoid, is a mathematically defined surface that approximates the shape of the Earth. It is an oblate spheroid, meaning it is slightly flattened at the poles and bulging at the equator due to Earth's rotation. The global ellipsoid is defined by two main parameters:

Semi-major axis (a): The equatorial radius of the ellipsoid.

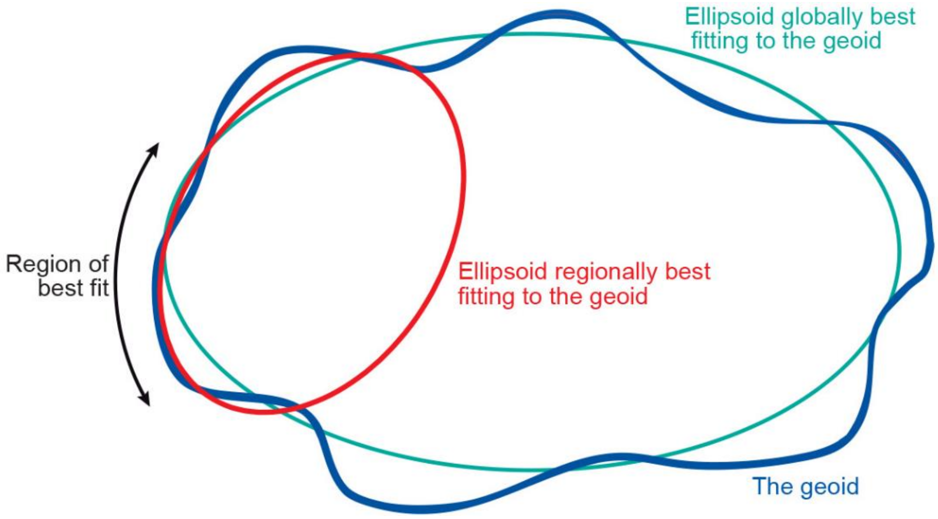
Flattening (f): A measure of how much the ellipsoid deviates from being a perfect sphere. It is defined as

$$\boldsymbol{f} = (\boldsymbol{a} - \boldsymbol{b})/\boldsymbol{a} \qquad \text{where b is the polar radius.}$$

WGS 84 (World Geodetic System 1984) is the most widely used global reference ellipsoid, especially in GPS and other satellite navigation systems. It is designed to provide a best-fit approximation of the Earth's shape globally. GRS 80 (Geodetic Reference System 1980) is another widely used global ellipsoid, often used in geodesy and mapping.

Local Ellipsoid

A local ellipsoid is similar to a global ellipsoid but is tailored to provide a better fit for a specific region or country. These ellipsoids are chosen to minimize discrepancies between the ellipsoid and the actual Earth's surface in a particular area. The parameters of a local ellipsoid are selected to closely match the Earth's surface in a specific geographic region. This is useful for local mapping and surveying projects where higher accuracy is needed than can be provided by a global ellipsoid. Clarke 1866 Ellipsoid, Everest 1830 Ellipsoid, Bessel 1841 Ellipsoid are some of local ellipsoids.



Source: ITC Department of Geo-information Processing (GIP)

Figure -1: Geoid, Global ellipsoid and Local ellipsoid

Geoid Undulation

The geoid undulation model specifically deals with the variations or "undulations" in the geoid relative to a standard reference global ellipsoid which is a mathematically defined, smooth surface approximating the Earth's shape. These deviations, known as geoid undulations or geoid heights, are fundamental in understanding the Earth's gravitational field and its impact on various scientific and practical applications.

These geoid undulations arise from irregularities in the Earth's mass distribution, such as mountain ranges, ocean trenches, and variations in crustal density. They are crucial for accurate geodetic measurements because they allow for the precise determination of true orthometric heights, which are the heights measured from the geoid. The geoid undulation model is thus indispensable in various fields, including geodesy, oceanography, and engineering, where precise elevation data are necessary.

With the rapid development of Global Navigation Satellite System(GNSS), the involvement of GNSS technology to the expansion of geodetic applications in surveying and engineering fields is increasing day by day. Though the GNSS has the ability of providing three-dimensional positioning (latitude, longitude and height) for a particular point in the earth surface, it determines the geometrical heights(ellipsoidal height), which cannot be used directly in the surveying and engineering activities. The ellipsoidal heights should be converted to Orthometric heights which are physically meaningful for above applications. The conversion of ellipsoidal heights which are obtained from GNSS data in to orthometric heights can be considered as a major application of geoid model. Other than that, understanding the geoid is vital for studying ocean circulation, sea level changes, and climate-related phenomena. The geoid undulation model helps in mapping the mean dynamic topography of the oceans. By studying variations in the geoid, scientists can infer information about the Earth's internal structures, such as mantle convection and tectonic activities, polar motion, crustal deformation which are the precursors for geo hazards like earthquakes, landslides, tsunamis etc.

When there is an accurate geoid model, it can be used to derive orthometric heights through calculation of geoid undulation. Defining an accurate, most suitable geoid model for an area is crucial to obtain precise measurements. Geoid models can be classified according to the area that they covered. There are global geoid models as well as local geoid models. There are large number of Global Geopotential models which are established and developed during last few decades such as Ohio State University model(OSU 91A), Earth Geopotential models(EGM) etc. EGM 1996, EGM 2008 and XGM 2019e_2159 are mostly used in the geodetic applications and several studies has done to evaluate the accuracy and the precision of the model.

b. Defining geoid model

There are several methods to define geoid model for a country, region or globe. Gravimetric method, geometric method, astro-geodetic, astro-gravimetric, transformation and hybrid methods are some of such methods. Gravimetric approach, Geometric method and the Hybrid approach are highly used in defining geoid models.

Gravimetric geoid modelling

Gravimetric geoid modeling is the most direct method of defining a geoid model, using measurements of the Earth's gravitational field. It relies on measurements of the Earth's gravitational field to compute the geoid's shape and height. There are basically three phases. In the first phase terrestrial, airborne and satellite gravity data will be collected. In terrestrial gravimetry, gravimeters are used to measure the Earth's gravity at specific points on the ground. These instruments detect small variations in gravitational acceleration caused by differences in the Earth's mass distribution. In areas where ground measurements are challenging, aircraft equipped with gravimeters are used to collect data. This method covers larger areas more quickly, though with slightly less precision than terrestrial methods. In the satellite gravimetry, satellites such as those in the GRACE (Gravity Recovery and Climate Experiment) mission are measured gravity anomalies from space. These satellites provide global data coverage, essential for areas like oceans and remote regions.

In the second phase, gravity anomalies are calculated. Gravity anomalies represent the difference between the observed gravitational acceleration and a theoretical reference model, such as the Earth's gravitational model (EGM). These anomalies reflect variations in Earth's density and mass distribution. In the third phase, the collected gravity anomalies are integrated using Stokes' formula to compute the geoid undulations. This involves integrating the gravity anomalies over the Earth's surface, which requires accounting for the Earth's curvature. Spherical harmonic analysis is used to model the gravity field mathematically. This method decomposes the Earth's gravitational potential into a series of spherical harmonics, allowing for a global and regional representation of the geoid. The final step involves calculating the geoid's surface by summing the contributions from all the spherical harmonics and adjusting for local gravitational variations. This provides a detailed map of the geoid undulations relative to a reference ellipsoid.

Geometric geoid modelling

The geometric method, also known as the GPS/leveling method, determines the geoid using geometric measurements, primarily from GNSS (Global Navigation Satellite System) and leveling techniques. GNSS provides accurate measurements of the ellipsoidal height (h) of points on the Earth's surface. These heights are relative to a reference ellipsoid, such as WGS84. Traditional leveling techniques, which involve measuring the difference in height between two points using a leveling instrument, provide orthometric heights (H). Orthometric heights are relative to the geoid and represent the actual elevation above mean sea level. Then geoid undulation (N) at any given point is determined using the formula :

$$\boldsymbol{N = h - H}$$

This formula calculates the difference between the ellipsoidal height (h) and the orthometric height (H). The resulting geoid undulation(N) represents the height difference between the geoid and the reference ellipsoid.

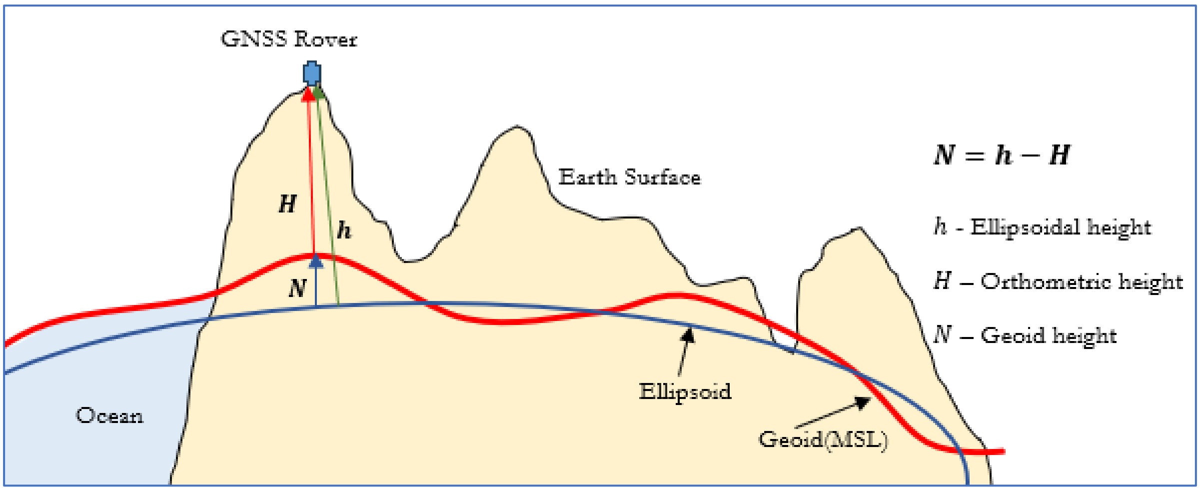


Figure- 2: Relationship between Orthometric height, ellipsoidal height and geoid height

Since GNSS and leveling data are often sparse and irregularly distributed, interpolation methods are used to estimate geoid undulations at unsampled locations. Techniques like kriging or polynomial fitting can be applied. Adjustments are made to account for systematic errors and inconsistencies in the data, ensuring a coherent and accurate geoid model.

Hybrid geoid modelling

The gravimetric method is more comprehensive and provides a global view of the geoid, as it incorporates data from various sources, including remote and oceanic regions. It is particularly valuable for scientific research, such as studying Earth's internal structure and mass distribution. But the existing global gravitational models such as EGM96, EGM2008, SGG-UGM-2, XGM2019 which are spherical harmonic models of Earth's gravitational potential shows discrepancy around 1.5m from independent levelling values for a particular location in Sri Lanka.

The geometric method, while more localized, offers high precision in areas with dense GNSS and leveling networks. It is especially useful for practical applications like engineering, surveying, and mapping, where precise height data is critical. But, carrying traditional levelling for each and every point in the surface is not practical as well as it is costly.

Both methods are complementary, and in modern geoid determination, they are often combined to produce the most accurate and detailed models. The gravimetric method provides a broad, global framework, while the geometric method refines and enhances this framework with high-resolution local data. This combination ensures that the resulting hybrid geoid model accurately represents the Earth's shape and gravitational field, supporting a wide range of scientific and practical applications. Since the accuracy of global geoid models are not sufficient for many surveying and engineering applications, most of the countries are taking attempts to establish own regional geoid models and most of them are hybrid geoid models. GSIGEO2011(Japan), AUSGEOID2020(Australia), EGG2015(Europe), GCG2016(Germany) are examples for such models which are regionally used for their surveying applications.

Literature Review

There are several global and local geoid undulation models have been developed by different countries all over the world. Different researches were carried out based on different algorithms and methods to obtain their required accuracy.

Richard and Wang in 1992, research on geoid undulation differences between Geopotential models was carried out to compare three geopotential models: OSU91A, GEM-T3, and GRIM4-C2. The models were evaluated by higher degree terms through comparisons with undulations at Doppler and GPS positioned stations. The undulation difference at Doppler stations was obtained as ± 1.57m with no significant difference between the models. A final comparison was made between geoid undulations implied by a Geosat 17-day cycle and undulations from the three models. The OSU91A model had best results with a difference standard deviation of ± 34 cm.

GEOID99 and G99SSS: 1-arc-minute geoid models for the United States were developed by Smith et al. in 2001 which were Hybrid model and purely gravimetric model respectively. G99SSS gravimetric model was referenced to the geocentric GRS 80 ellipsoid and Hybrid

GEOID99 model encompasses all the information of G99SSS and vertical datum information of 6169 GPS- derived NAD 83. GEOID96 had the ability to convert absolute heights at the ±5.3-cm level and GEOID99 works with the latest GPS BMs at the ±4.6-cm level.

With the high accuracy requirement, geoid modelling for Japan is also critical task. There are several studies were carried out to improve high-resolution gravimetric geoid model from EGM2008 combined with local terrestrial gravity data. In the GOCE GGMs, the residual gravity anomalies were accomplished by Kriging on a 1x 1.5 grid and Stoke's integral formula has been used and modified Stoke's Kernal was applied to minimize the truncation errors. In comparison with the previous geoid model for Japan at 816 GPS/levelling points, there was an improvement in the standard deviation from ±8.3 cm to ±7.5 cm which was obtained from using of the integral formulae to handle the topographic effects on gravity. With dense coverage and accurate gravity data over Japan, the model accuracy will be further increased. (Odera and Fukuda, 2012)

In 2016, GSIGEO2011 was developed as a new hybrid geoid model by improving JGEOID2008 with GNSS/levelling geoid undulation at 971 sites by the Least Squares Collocation (LSC) method by Diego . The developed model was able to achieved standard deviation of 1.8cm which is same as the third-order levelling surveys. (Miyahara, 2014)

A Gravimetric Geoid Model for Argentina was developed by combining the latest global geopotential models (GGMs), along with detailed digital terrain models (DTMs) and terrain corrections were applied. Kriging method was used to decrease the residual gravity anomalies and resultant grid was applied in Stoke's integral using FFT approach. The model used 1904 benchmarks with both ellipsoidal heights and orthometric heights and able to achieved up-to 10cm accuracy.

A geoid model for the Western India was developed in 2018 . Firstly, gravimetric geoid was developed using terrestrial gravity and satellite altimetry derived marine gravity data. Then the gathered data were connected with best fitted Global geopotential model for their region using Remove-Compute-Restore technique which involves Fast Fourier Transformation and optimized Stoke's Kernal. Using this gravimetric model, accuracy level of 14cm RMSE was achieved using this model and then the model was subsequently fitted by Least Squares Collocation (LSC method) with GNSS observation data of 1st order benchmarks at 39 locations and were able to achieve 7cm RMSE for 13 locations.(Singh and Srivastava, 2018).

Detailed procedure for determining local gravimetric- geometric geoid model of an area was presented by Eteje Sylvester O. et al., (2018) which gives Gravimetric geoid heights computation using the modified Stokes Integral formula and several geometric geoid surfaces fitting methods. Plane surfaces, Bi-linear and second-degree surfaces and polynomial surfaces are deeply described. Also, observation equation method of least square adjustment was introduced for fitting of geometric geoid surface to a set of undulation values to compute model parameters.

Several attempts were taken to develop geoid model for Sri Lanka time to time. It can be identified high variation in the topography in the central region of the country which exceed 1800m in height. There is a negative Bouguer anomaly reaching a minimum of 50 m Gal over this region. Therefore, a relative geoid model for central of Sri Lanka has been developed adding contributions due to Bouguer anomaly and topographical features of that area. Instead of using classical gravimetric method to obtain the topographic component, Fast Fourier Algorithm has been connected which is based on Stoke's Integral Formula. This work was shown that the relative geoid of central height of Sri Lanka is entirely positive with maximum of 5.9m close at Nuwara Eliya. (Tantrigoda,1995)

Sri Lanka is located in an area where the lowest elevation with respect to a geocentric reference ellipsoid. Suitability of EGM2008 for the Sri Lanka and its behavior was examined using "fill-in" methodology used in EGM2008 performs versus observed data. It shows -1.75m bias between the GPS-levelling and EGM2008 and ±0.184m accuracy was obtained using 207 GPS levelling points after rejection 15 outliers. Also, after rejection of outliers in the terrestrial gravity data, it was yielded standard deviations of ±6.743 mGal for 20 GPS-coordinated gravity points on fundamental benchmarks, ±14.704 mGal for 42 gravity points on fundamental benchmarks. (Abeyratne et al., 2009)

To improve EGM2008 Geoid undulation values to Sri Lanka, quasi geoid model was introduced. Polynomial surface fitting method was with 37 benchmark points and 10 BM independent points which are not used in the model were used to validate the model. Matlab software was used to perform the model parameter calculation with third order polynomial. The prepared quasi model was achieved ±0.05m accuracy for 20 BMs out of 47 and another 8 BMs were in the accuracy range of ±0.05m-0.10m. (Nanayakkara, 2016)

However, it is important to note that using a small number of model points may not accurately represent the actual variation of the phenomenon. This is because the model points are used to estimate the accuracy of the model within a specific range, but they may not capture the full range of variability in the phenomenon. Therefore, it is recommended to use a larger number of model points and to validate the model using independent points whenever possible.

Problem Statement

GNSS technology gives position of any point quickly, but it is difficult to obtain orthometric height directly from those data. If the geoid undulation can be obtained precisely and easily from GNSS observations, it will give a way to determine orthometric heights easily. Though there are several attempts have been taken, still there is no geoid undulation model which is fit with the required accuracy standards for Sri Lanka. As a results, when the orthometric heights are needed for an application, time consuming differential levelling methods are adopted. So, developing an accurate geoid undulation model is very much essential and timely needed to obtain orthometric height directly using GNSS observations.

Objectives

The main objective of this work was to develop accurate geoid undulation model for Sri Lanka to determine orthometric height of any given point which has the ellipsoidal height. The model was further developed to a Hybrid geoid model by integrating the gravity measurements to enhance the required accuracy. By developing a user-friendly Graphical interface, user will able to obtain orthometric height by feeding north, east coordinate and ellipsoidal height.

Methodology

a. Flow of Methodology:

This research work was carried to develop accurate geoid model for Sri Lanka. Developing the geoid undulation model was basically consisted as two sections. In the first section geoid undulation for selected control points were calculated. Calculation of geoid undulation was also carried out as two stages. In the first stage, normal(geometric) geoid undulation was computed using orthometric height which were obtained from levelling and the ellipsoidal heights which were obtained from GNSS observations. In the second stage gravity undulations for each point were calculated. To calculate the gravity undulation, EGM2008 model and XGM9019 gravity model were used with the North, East coordinates and the ellipsoidal heights. Then to develop Hybrid geoid model, difference between normal undulation and gravity undulation(known as Offset) was calculated and the offset was modeled.

In the second section of the developing Geoid model was the surface fitting for the calculated undulation values in the first section. There are several methods which are applicable for the surface fitting such as polynomial regression, kriging interpolation, Bezier Surface fitting, IDW etc . By considering the previously obtained results of the other researches, polynomial fitting method was used with the observation equation method of least square adjustment for model parameter determination. Once the parameters were determined for the selected data set, those parameters were used to create the model and to calculate the undulation of unknown point.

The suitable polynomial order for the model was selected by checking the RMSE value for the same data set with increasing order. Order of the polynomial which had the minimum RMSE value was order five and selected to proceed the work.

Once the geoid undulation was modeled in to a surface, undulation value of any point can be obtained from the model. To derive the orthometric height for a particular unknown point, modeled undulation value was combined with the ellipsoidal height.

To increase the accuracy of the result, Hybrid model was developed. To calculate the undulation at a particular point, the modeled value of the offset model at that point was combined with gravity data which are obtained from the EGM2008 and XGM2019. Then the obtained gravity undulation was combined with the ellipsoidal height to obtain the orthometric height.

The whole procedure was completed for 759 data points and the model was independently checked for 72 data points.

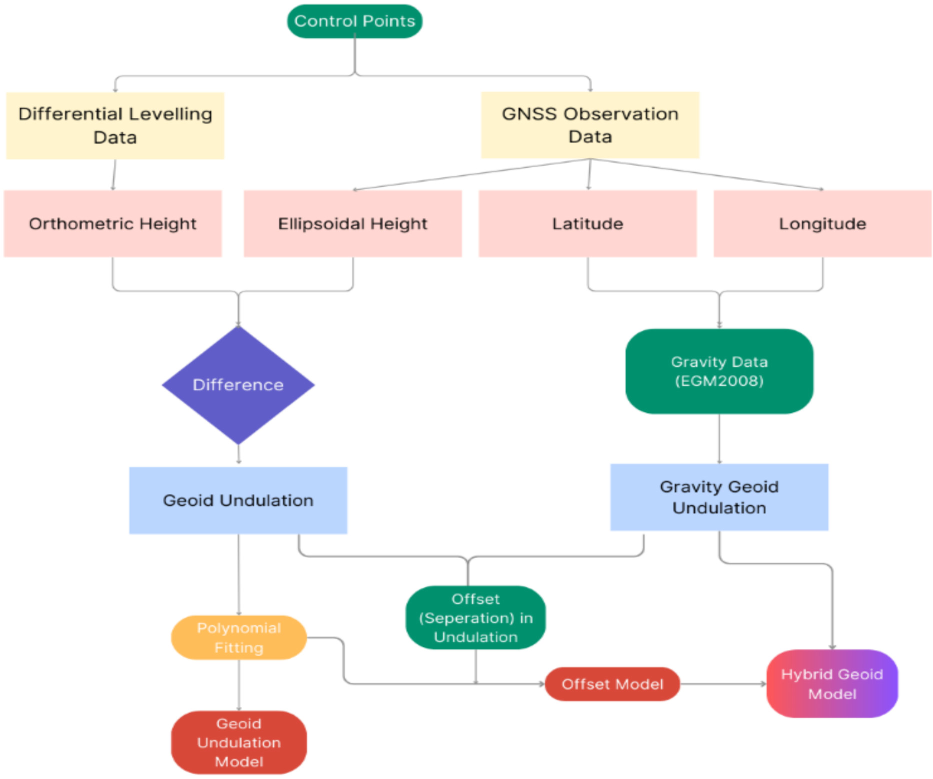


Figure-3: Methodology Diagram

b. Theoretical background :

Polynomial surface fitting method

Polynomial surface fitting is a statistical technique used to model a relationship between a dependent variable and two or more independent variables using a polynomial equation. In the context of geospatial data, it is often used to approximate a surface, in this case a geoidal surface, by fitting a polynomial function to observed data points.

In generally, A polynomial function in two variables x and y can be expressed as:

$$N(x,y) = \sum_{j=0}^{n}\sum_{i=0}^{n} [a_{ij}x^i y^j] \;\; ; i+j \le n$$

Where

a_{ij}= Unknown coefficient of polynomial

n = The degree of the polynomial

x,= Independent variables(in this case North East coordinates)

$N(x,y)$ = Dependent variable(in this case geoid undulation or offset between geoid undulation and gravity undulation)

The degree of the polynomial is determined by the highest power of the independent variables in the equation. For example, in a quadratic polynomial surface (second degree), the highest terms would be x^2, y^2, xy.

According to the residual values suitable polynomial order was fifth order, which can be express as;

$$\sum_{i=0}^{n} N_i = a_0 + a_1E_i + a_2N_i + a_3E_i^2 + a_4N_i^2 + a_5E_iN_i + a_6N_i^3 + a_7E_i^3 + a_8E_iN_i^2 + a_9E_i^2N_i + a_{10}N_i^4 + a_{11}E_i^4 + a_{12}E_iN_i^3 + a_{13}E_i^2N_i^2 + a_{14}E_i^3N_i + a_{15}E_i^5 + a_{16}N_i^5 + a_{17}E_i^4N_i + a_{18}E_iN_i^4 + a_{19}E_i^2N_i^3 + a_{20}E_i^3N_i^2$$

where n is the number of observations.

The fitting process involves determining the coefficients $a_0, a_1, a_2, \ldots a_{20}$ that minimize the difference between the observed data points and the polynomial surface. This is usually done using a method called Least Squares which minimizes the sum of the squares of the differences between the observed and predicted values.

To apply Least square adjustment, polynomial equation is generated for each model point. Then, they are arranged in an observational matrix :

$$N_{nx1} = A_{n\times m} \times X_{m\times 1} + v_{n\times 1}$$

Where:

$N_{n\times 1}$= Observation matrix of Undulation

$A_{n\times m}$= Coefficient matrix

$X_{m\times 1}$= Parameter matrix

$v_{n\times 1}$= Residual matrix

n = Number of Observations

m = Number of Parameters

Limitations of Polynomial fitting method

- High-degree polynomials can lead to overfitting, where the model fits the noise in the data rather than the underlying trend.
- May not capture non-polynomial trends or features in the data.
- Sensitive to outliers, which can distort the fitted surface.

Polynomial surface fitting is a versatile tool in data analysis and modeling, but careful consideration of the degree of the polynomial and the nature of the data is essential for obtaining meaningful results.

Least Square adjustment

After formation of the observational matrix, least square method was used to find the value of parametric matrix.

The least squares method is a mathematical approach used to find the best-fitting line or curve to a given set of data points which is widely used in regression analysis, curve fitting, and data modeling. It minimizes the sum of the squares of the residuals between the observed data points and the values predicted by the model. In the context of polynomial surface fitting, if N_i is the observed value and $\widehat{N_i}$ is the predicted value at the same point, then the residual r_i is given by:

$$r_i = N_i - \widehat{N_i}$$

The least squares method aims to minimize the sum of the squared residuals. This sum, denoted by SSR, is given by:

$$SSR = \sum_{i=1}^{n}(N_i - \widehat{N_i})^2$$

To find the adjusted parameters X of the model that minimize the SSR,

$$\hat{X} = (A^T PA)^{-1} A^T P\, N$$

can be used.

Where:

N = Observation matrix of Undulation

A = Coefficient matrix

A^T = = Transpose of Coefficient matrix

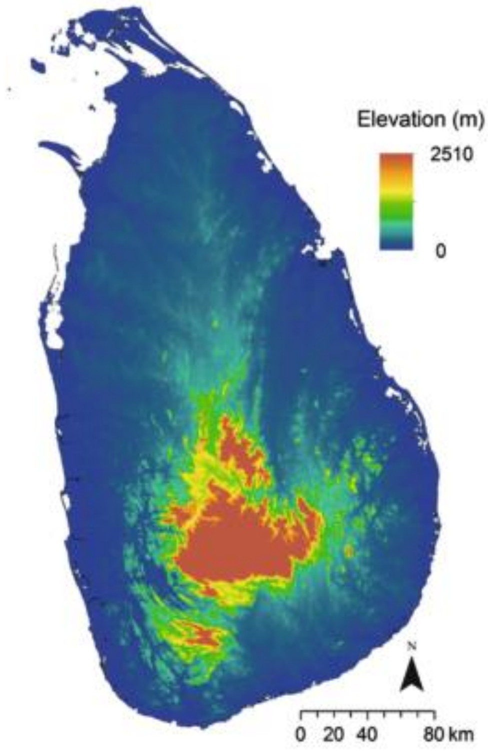
$\hat{X}$ = Adjusted Parameter Matrix

P = Weight matrix based on the precision of the observation

Once the adjusted parameter values of the model are obtained, they can be used to find the undulation value of unknown position which has North, East coordinate.

Study Area

To develop the normal geoid model and the hybrid geoid model, the study area was selected as whole Sri Lanka which has approximately 65,610 square kilometers of land located between latitudes 5°55′–9°51′ N and longitudes 79°41′–81°53′ E in the Indian Ocean. The most prominent feature is the central highlands, which include a range of mountains, plateaus, and valleys. The highest peak, Pidurutalagala, reaches an elevation of 2,524 meters above sea level. The central highlands are characterized by steep escarpments and deep valleys, making the area rugged and hilly. Surrounding the central highlands are coastal plains that are generally flat and extend towards the coastline. These plains are more extensive in the northern and eastern regions of the island. When considering the geological composition which is highly influenced to the gravity measurements, Sri Lanka's geological history dates back to the Precambrian era, with over 90% of the rocks being crystalline metamorphic rocks. These rocks are part of the ancient Gondwana supercontinent. The island's geological formations include a mix of igneous, sedimentary, and metamorphic rocks. Sri Lanka is an area of notably low gravity compared to the Earth's average gravitational field. The gravity value in this region deviates around 9.77 m/s^2.



Source: Agricultural Research for Sustainable Food Systems in Sri Lanka

Figure-4: Elevation variation in Sri Lanka

Results and Discussion

The results of the geometric model and the hybrid model for the model points and the independent checking points were mentioned in below:

Table 1: Summery of results from model

Residual Range	Number of model points within the mentioned range out of 759 points		Number of points within the mentioned range out of 72 independent checking points	
	Using Geometric Geoid Model	Using Hybrid Geoid Model	Using Geometric Geoid Model	Using Hybrid Geoid Model
$0 - 5\ cm$	91	293	10	29
$5 - 10\ cm$	122	230	6	15
$10 - 20\ cm$	153	174	14	21
$20 - 30\ cm$	123	40	14	6
$30 - 50\ cm$	149	17	20	1
$50 - 80\ cm$	74	5	8	0
$80 - 100cm$	20	0	0	0
$> 100cm$	27	0	0	0
Total	759	759	72	72

Table 2: Summery of results from model as a Percentage

Residual Range	Percentage of model points within the mentioned range out of 759 points		Percentage of points within the mentioned range out of 72 independent checking points	
	Using Geometric Geoid Model	Using Hybrid Geoid Model	Using Geometric Geoid Model	Using Hybrid Geoid Model
$0 - 5cm$	11.99%	38.60%	13.89%	40.28%
$0 - 10\ cm$	28.06%	68.91%	22.22%	61.11%
$0 - 20\ cm$	48.22%	91.83%	41.67%	90.28%
$0 - 30\ cm$	64.43%	97.10%	61.11%	98.61%
$0 - 50\ cm$	84.06%	99.34%	88.89%	100.00%
$0 - 80\ cm$	93.81%	100.00%	100.00%	
$0 - 100cm$	96.44%			

a. Contour base model point selection

When selecting the model points for the geoid undulation model, a common approach is to choose the points based on a contour pattern, ensuring that each height range is represented. This can be particularly beneficial in highly variable terrain, such as the hilly regions of Sri Lanka, as it allows for the accurate interpolation of the geoid undulation across the entire elevation spectrum. However, in areas with limited data availability, this point selection method may have some drawbacks. In the hill country of Sri Lanka, the majority of the elevation data points tend to be concentrated along the main roads. This can result in a significant lack of data points at the higher elevations (hilltops) and lower elevations (valleys), which are important for the accurate interpolation of orthometric heights.

b. Analysis of Undulation pattern in Sri Lanka

The pattern of Geoid undulation was varying with the topography of the country. The western side of Sri Lanka shows significantly lower geoid undulation values. This suggests the western coastal regions have a relatively flatter geoid surface compared to the rest of the island. The hill country region of Sri Lanka, which exhibits a more varied and higher range of geoid undulation values. The eastern side shows higher geoid undulation values compared to the western side

The differences in the geoid undulation between the eastern and western sides of the island can be attributed to factors. If considered topographic influence, the western side is characterized by relatively flat coastal plains, while the eastern side is dominated by the central hill country with higher elevations.

The differences in the underlying geology and crustal structure between the eastern and western regions of Sri Lanka can lead to variations in the local gravity field. These gravity field variations can influence the geoid undulation, resulting in potential differences in the magnitude and patterns of the geoid undulation between the two sides of the island. The proximity to the coastline can also influence the geoid undulation, as factors such as ocean depth and seafloor topography can affect the local gravity field. The western coastal regions of Sri Lanka may exhibit different geoid undulation characteristics compared to the eastern coastal areas due to these coastal effects.

c. Modeling the Geoid Undulation rather than Modeling Terrain Elevation

Modeling the geoid undulation directly, rather than just modeling the elevation, is a more effective approach for determining accurate orthometric heights in Sri Lanka. There are few key reasons to follow this approach, as Terrain variability. The terrain in Sri Lanka is highly variable, with mountainous regions, valleys, and diverse geological features. Modeling the elevation alone would be challenging and may not capture the complexities of the landscape accurately. The geoid, is a more consistent and well-defined reference surface that is less affected by the rapid changes in terrain.

Another key point is the effects of gravity Geoid is a model of the Earth's gravity field, which is a fundamental factor in determining orthometric heights. Variations in gravity due to changes in density, mass distribution, and other geophysical factors can significantly affect the relationship between ellipsoidal and orthometric heights. By modeling the geoid undulation, these gravity effects can be more accurately accounted for.

GNSS-derived ellipsoidal heights are measured with respect to the reference ellipsoid, which is an idealized mathematical surface. To obtain meaningful orthometric heights, which are measured from the geoid, a geoid undulation model is necessary to bridge the gap between the two height systems. This allows for a seamless conversion of heights. Modeling the geoid undulation provides a more consistent and accurate reference surface for height determination across the entire country.

Conclusion and Recommendation

The development of an accurate and reliable geoid undulation model is essential for Sri Lanka to enable precise positioning and elevation determination across the island. This study has demonstrated the feasibility of creating such a model using a hybrid approach that combines terrestrial gravity data or satellite-derived gravity information with the geoid heights. A key finding is the importance of using high-quality, reliable source data for the model development. The accuracy and consistency of the differential levelling measurements, GNSS surveying and data processing techniques, terrestrial gravity measurements, as well as the precision of the satellite-derived gravity data, are critical inputs that directly impact the reliability of the final geoid undulation model. Relying solely on gravity model data, without incorporating actual terrestrial gravity observations, was found to produce less accurate results, particularly in regions with complex terrain such as the central hill country of Sri Lanka.

The accuracy of the orthometric height calculated using the geoid undulation model is primarily influenced by a few critical factors. The quality, resolution, and reliability of the geoid undulation model makes a high influenced for it. Errors or uncertainties in the geoid undulation values will directly translate into errors in the final orthometric height calculation. Additionally, the accuracy of the GNSS-derived ellipsoid height measurement is also a critical factor, as any errors in the ellipsoid height will contribute to the overall error in the orthometric height.

Another important consideration is the complexity of the terrain. In areas with steep or irregular terrain, the geoid undulation can vary rapidly over short distances, and the geoid model may not be able to accurately capture these fine-scale variations, leading to larger errors in the orthometric height calculation. Furthermore, local variations in the Earth's crustal density can cause small-scale distortions in the geoid surface that are difficult to model accurately, which can also introduce errors in the orthometric height derivation.

The hybrid approach, provided the most reliable and accurate geoid undulation model for Sri Lanka. This combined model performed better than models based on only one data source, whether terrestrial or satellite data alone. It delivered improved accuracy across both the flat coastal areas and the hilly inland regions of the country.

Furthermore, it will be essential to continue expanding and refining the network of terrestrial gravity data collection in Sri Lanka. Regularly updating the model with new gravity observations, as well as incorporating advancements in satellite-based gravity measurement techniques, will ensure the geoid undulation model remains accurate and useful for Sri Lanka's surveying and mapping needs over time.

References

Abeyratne, P.G.V. & Featherstone, Will & Tantrigoda, Dhammika. (2010). Assessment of EGM2008 over Sri Lanka, an area where 'fill-in'data were used in EGM2008. Newton Bull.. 4.

Edirisinghe, Dinithi & Welikanna, Duminda & Periyandy, Thunendran & Bandara, Ranmalee. (2024). Development of a hybrid geoid model using a global gravity field model over Sri Lanka. Journal of Applied Geodesy. 10.1515/jag-2023-0092.

Miyahara B, Kodama T, Kuroishi Y. Development of new hybridgeoid model for Japan, " GSIGEO2011" Development of newhybrid geoid model for Japan, " GSIGEO2011" Basara MIYAHARA,Tokuro KODAMA, Yuki KUROISHI. Bull Geos Inf Auth Japan2014;62:11−20

Nanayakkara N.S.W.S.P., (2016), Determination of the Quasi geoid of Sri Lanka based on EGM2008.
http://dlib.pdn.ac.lk/handle/1/5609

Odera, Patroba & Fukuda, Yoichi. (2012). Towards a precise geoid model over Japan, Gravity, Geoid and Height Systems October 9 – 12, 2012, Venice, Italy

Pinon, D. A. (2016). Development of a Precise Gravimetric Geoid Model for Argentina. March, 182. https://researchrepository.rmit.edu.au/discovery/fulldisplay?docid=alma9921864076501341&context=L&vid=61RMIT_INST:ResearchRepository&lang=en&search_scope=Research&adaptor=Local Search Engine&tab=Research&query=any,contains,Development of a Precise Gravime

Rapp, R.H., Wang, Y.M. Geoid undulation differences between geopotential models. Survey Geophysics 14, 373–380 (1993). https://doi.org/10.1007/BF00690565

Smith, Dru & Roman, Daniel. (2001). GEOID99 and G99SSS: 1-arc-minute geoid models for the United States. Journal of Geodesy. 75. 469-490. 10.1007/s001900100200.

Suresh Singh and Rajiv Srivastava, (2018), Development of Geoid Model- A Case Study on Western India, FIG Congress 2018 Embracing our smart world where the continents connect: enhancing the geospatial maturity of societies Istanbul, Turkey, May 6–11, 2018, India

Sylvester Okiemute, Eteje & Olujimi, Oduyebo & Sunday Adekunle, Olulade. (2018). Procedure for the Determination of Local Gravimetric-Geometric Geoid Model. International Journal of Advances in Scientific Research and Engineering. Volume 4. 206-214. 10.31695/IJASRE.2018.32858.

Tantrigoda, D.A. (1995). Relative Geoid of Central Sri Lanka. Vidyodaya Journal of Science, Volume 5(1), 47-55